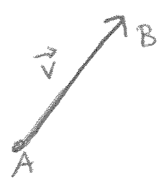
12.2 Vectors

A vector is a quantity that has both magnitude and direction.

We usually express it as an arrow, where the length of the arrow represents magnitude and the arrow points in the direction of the vector.

Example:

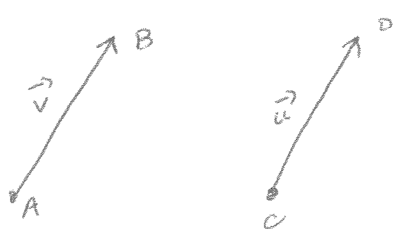


We call A the tail of $\vec{v}$ and B the tip of $\vec{v}$.

We sometimes write $\vec{v} = \overrightarrow{AB}$.

If two vectors have the same magnitude and direction, we say they are equal. (even though they may be placed at different locations.

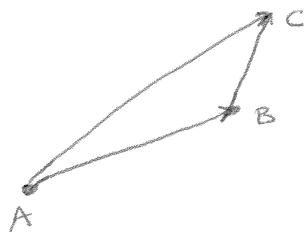
Example:



The zero vector has length 0 and it is the only vector with no direction.

Adding Vectors

Let say we have a particle moving from A to B and then moving from B to C. What is the resulting displacement?



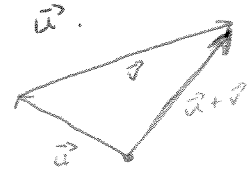
The resulting displacement is $\vec{AC}$ and we say $\vec{AB} + \vec{BC} = \vec{AC}$.

In general when adding vectors $\vec{u}$ and $\vec{v}$ there are two ways to do thi

I. Triangle Law

Step 1. Draw $\vec{u}$

Step 2. Draw $\vec{v}$ such that the initial point of $\vec{v}$ is in the terminal point of $\vec{u}$.

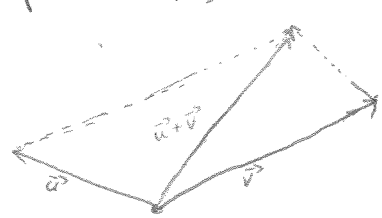


Then the vector from initial pt of $\vec{u}$ to terminal pt of $\vec{v}$ is $\vec{u}+\vec{v}$.

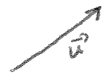
II. Parallelogram Law

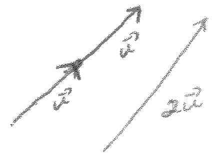
Step 1: Draw both $\vec{u}$ and $\vec{v}$ from the same initial pt.

Step 2: Draw a parallelogram with sides $\vec{u}$ and $\vec{v}$.



Scalar Multiplication

Let $\vec{u}$ be the vector . What is $2\vec{u}$?

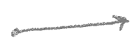


What about $3.5\,\vec{u}$? ...

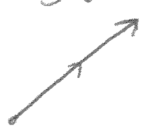
What is $-\vec{u}$?

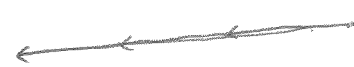
In general, if c is a scalar (a number) and $\vec{u}$ is a vector then

$c\vec{u}$ is
- the vector with length $|c|$ times the length of $\vec{u}$, direction is the same as $\vec{u}$; if $c>0$
- the vector with length $|c|$ times the length of $\vec{u}$, direction opposite to $\vec{u}$. if $c<0$.

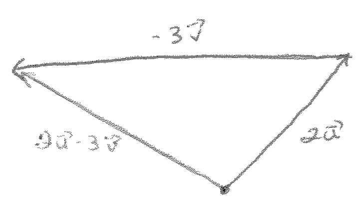
Example: let $\vec{u}$ be , $\vec{v}$ be ,

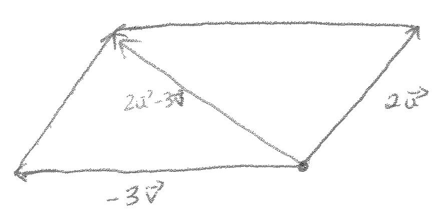
Draw $2\vec{u} - 3\vec{v}$.

① $2\vec{u}$ is .

② $-\vec{v}$ is , so $-3\vec{v}$ is 

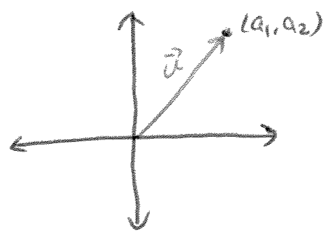
So using triangle law, using parallelogram law,



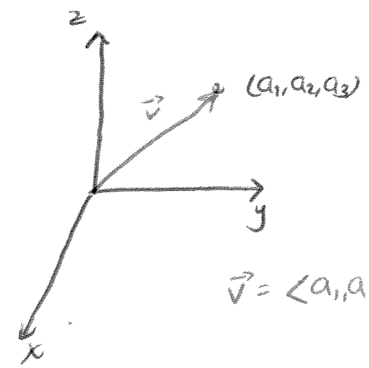


Using Coordinates

It is very useful to treat vectors in our usual coordinate system.

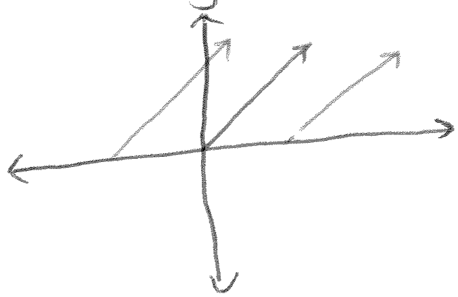


$\vec{u} = \langle a_1, a_2 \rangle$



$\vec{v} = \langle a_1, a_2, a_3 \rangle$

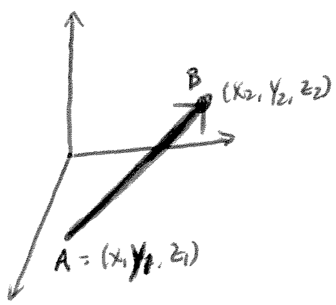
(Optional) The following vectors are the same.



Important:

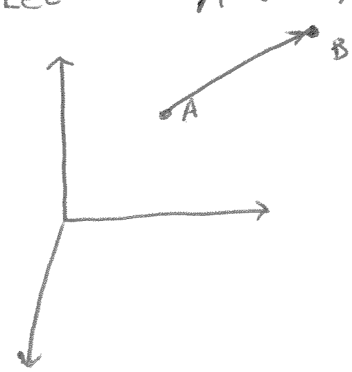
- Given two points $A = (x_1, y_1, z_1)$ and $B = (x_2, y_2, z_2)$, the vector

  $\vec{AB} = \langle x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1 \rangle$



- Example:

  Let $A = (1, 2, 3)$, $B = (-2, 7, 9)$



$\vec{AB} = \langle -2-1,\ 7-2,\ 9-3 \rangle$

$= \langle -3, 5, 6 \rangle.$

Important:

The magnitude or length of a vector a is denoted by $|a|$. ~~and the~~

In 2-dim: if $a = \langle a_1, a_2 \rangle$ then $|a| = \sqrt{a_1^2 + a_2^2}$

In 3-dim: if $a = \langle a_1, a_2, a_3 \rangle$ then $|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Addition, Substraction and scalar multiplication:

Let $a = \langle a_1, a_2 \rangle$, $b = \langle b_1, b_2 \rangle$ then

2-dim: $a+b = \langle a_1+b_1, a_2+b_2 \rangle$; $a-b = \langle a_1-b_1, a_2-b_2 \rangle$; $ca = \langle ca_1, ca_2 \rangle$

3dim: $a+b = \langle a_1+b_1, a_2+b_2, a_3+b_3 \rangle$; $a-b = \langle a_1-b_1, a_2-b_2, a_3-b_3 \rangle$; $ca = \langle ca_1, ca_2, ca_3 \rangle$

Example: $a = \langle 3, -1, 7 \rangle$, $b = \langle 2, 4, -2 \rangle$

find $|a|$, $|b|$, $a+b$, $3b-2a$.

Properties of vectors

1. $a+b = b+a$
2. $a+(b+c) = (a+b)+c$
3. $a+0 = a$
4. $a+(-a) = 0$
5. $c(a+b) = ca+cb$
6. $(c+d)a = ca+da$
7. $(cd)a = c(da)$
8. $1 \cdot a = a$.

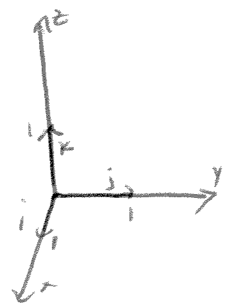
Claim: You can get any 3-dim vector using $\langle 1,0,0\rangle$, $\langle 0,1,0\rangle$ and $\langle 0,0,1\rangle$.

Let $a=\langle \quad \rangle$, then

$$a = \_\langle 1,0,0\rangle + \_\langle 0,1,0\rangle + \_\langle 0,0,1\rangle$$

We call them:

$i=\langle 1,0,0\rangle$, $j=\langle 0,1,0\rangle$, $k=\langle 0,0,1\rangle$.



Important:

A unit vector is a vector of length 1. For example, i, j and k. (Verify).

Let a be a vector, then $u=\left(\frac{1}{|a|}\right)a$ is a vector in the same direction as a and length 1.

Example: Find an unit vector in the direction of $3i-2j+6k$.

Let $a=\langle 3,-2,6\rangle$, then

$$|a|=\sqrt{3^2+(-2)^2+6^2}=\sqrt{49}=7.$$

Hence $u=\frac{1}{7}a=\langle \frac{3}{7}, \frac{-2}{7}, \frac{6}{7}\rangle$ has magnitude 1 and direction same as a.